Supplementary Material

Parameterisation of the Host Module

Parameters for the host module of LRVF are determined using literature values, values obtained as part of the HEALTHY FUTURES project and discussion with local experts. Based on this data, the livestock host represents both cattle and sheep and the following population properties are assumed:

- Cattle:sheep ratio = 33:66
- Cattle – bull:cow ratio = 30:70
- Sheep – ram:ewe ratio = 40:60

For a summary of all host-related parameters see Table 1.

Basal mortality rate

For the basal per capita mortality rate we use the inverse of the natural livestock lifespan. We assume the same natural mortality rate for both neonatal and adult livestock. We use HF D3.2 lifespan ranges and assume they are equally distributed about the mean in order to calculate average lifespans.

Livestock lifespan = $0.3\times0.33\times6.5\text{y} + 0.7\times0.33\times9.5\text{y} + 0.4\times0.66\times2\text{y} + 0.6\times0.66\times 6\text{y} = 5.742\text{y}$

$$d_x = d_y = \frac{1}{365\times5.742} = 4.77\times10^{-4}$$

Maturation rate

Use inverse of time at which infant livestock is no longer considered neonatal.

Neonate maturation age = $(1/3)\times(4/12)\text{y} + (2/3)\times(2/12)\text{y} = 2/9\text{y}$

$$m = \frac{1}{365\times(2/9)} = 1.23\times10^{-2}$$

Rate of infection

The following derivation is based on Keeling and Rohani (2008). We consider a susceptible host individual that receives K vector bites per unit time (δt). This number of bites per host will depend on the number of mosquitoes per host (assume neonatal host in this example) in the biting stage of the gonotrophic cycle such that $K = \kappa Z/X$ (assuming a linear dependence). The number of infected bites from infected mosquitoes in the biting stage of the gonotrophic cycle, Z_I, will therefore be $\kappa Z_I/X$. The probability of infection resulting from a bite is s. Hence the probability (denoted by $1 - \delta q$) of an individual host escaping infection following $(\kappa Z_I/X)\times\delta t$ contacts is

$$1-\delta q=(1-s)^{\frac{\kappa Z_I}{X}\delta t}.$$

Therefore, the probability of infection is δq.

If we define $\beta=-\kappa\log(1-s)$ we can write the probability of transmission in a small time interval as

$$\delta q=1-e^{-\frac{\beta Z_I}{X}\delta t}$$

To translate this probability to a rate of transmission we expand the exponential (i.e. $e^x=1+x+\frac{x^2}{2!}+\cdots$), divide both sides by δt and take the limit of $\delta q/\delta t$ as $\delta t\to 0$. This gives

$$\frac{dq}{dt}=\beta\frac{Z_I}{X}$$

as the transmission rate per susceptible. We multiply by the susceptible population to determine the rate of transmission for the entire susceptible population (e.g. X_I) to give

$$\frac{dX_I}{dt}=\beta\frac{Z_I}{X}X_I$$

For our model we need to define $\beta=-\kappa\log(1-s)$ for both *Aedes* and *Culex* vectors. We assume the same susceptibility for both age categories:

$$s=1/3\times 0.1+2/3\times 0.2=1/6$$

Assume κ is the same for both age groups. $\kappa=K/(Z/X)$, i.e. the total no of daily bites divided by the total number of mosquitoes in the biting stage of the gonotrophic cycle per host. This translates as the per capita biting rate (temperature dependent).

$$\kappa=\begin{cases} LBI\dfrac{T-T_g}{D_g+T-T_g}, & T>T_g \\ 0, \text{ otherwise} & \end{cases}$$

where LBI is the Livestock Blood Index, an indicator that informs us the proportion of mosquito bites assumed to be on cattle or sheep (rather than other mammals). We assume $LBI=0.005$ for neonatal and adult livestock being bitten by *Aedes* mosquitoes and $LBI=0.25$ for neonatal and adult livestock being bitten by *Culex* mosquitoes.

$(T-T_g)/(D_g+T-T_g)$ describes the gonotrophic cycle rate of the mosquito dependent on temperature, T, where T_g is a temperature threshold and D_g a degree-day threshold. This functional form includes the time taken for a blood meal to be taken (1 day, independent of temperature) and the time for egg development ($D_g/(T-T_g)$ where T represents temperature).

We define the following quantities based on Detinova (1962):

$$D_g = \begin{cases} 65.4, & R < 10 \\ 37.1, & R > 10 \end{cases}$$

$$T_g = \begin{cases} 4.5, & R < 10 \\ 7.7, & R > 10 \end{cases}$$

Therefore,

$$\beta_x^A = \beta_y^A = \begin{cases} -0.005 \dfrac{T - T_g}{D_g + T - T_g} \log\left(1 - \dfrac{1}{6}\right) = 0.05 \times 0.182 \times \dfrac{T - T_g}{D_g + T - T_g}, & T > T_g \\ 0, \ \text{otherwise} \end{cases}$$

$$\beta_x^C = \beta_y^C = \begin{cases} -0.25 \dfrac{T - T_g}{D_g + T - T_g} \log\left(1 - \dfrac{1}{6}\right) = 0.25 \times 0.182 \times \dfrac{T - T_g}{D_g + T - T_g}, & T > T_g \\ 0, \ \text{otherwise} \end{cases}$$

Incubation parameter

Inverse of 3.5-day latent period (Turell et al. 1985, Gaff et al. 2007, Niu et al. 2012).

$$\sigma_x = \sigma_y = \frac{1}{3.5} = 2.86 \times 10^{-1}$$

Recovery rate

Inverse of 6.5-day infectious period (Bird et al. 2009, Nfon et al. 2012).

$$\gamma_x = \gamma_y = \frac{1}{6.5} = 1.54 \times 10^{-1}$$

Infection-induced mortality probability

The probability of dying due to RVF infection before recovering based on case fatality rates (Bird et al. 2009).

$$\rho_x = \frac{1}{3} \times 0.45 + \frac{2}{3} \times 0.85 = 7.17 \times 10^{-1}$$

$$\rho_y = \frac{1}{3} \times 0.055 + \frac{2}{3} \times 0.095 = 8.17 \times 10^{-2}$$

Crude birth rate

Births match adult livestock deaths (inverse of lifespan):

$$b = d_y$$

Crude adult import rate

Defined based on the unique (for feasibility) steady state Y^*, i.e. the externally maintained adult livestock population:

$$c = \frac{Y^*\left(d_y(d_x+m)-mb\right)}{(d_x+m)} = \frac{Y^* d_y d_x}{(d_x+m)}$$

Vector Model Specification

Mature vectors

$Z_X^{t,g,e}$ is a vector population in infection state *X*, at time t, gonotrophic stage *g* and extrinsic incubation stage *e*. Infection categories are *S* (susceptible), *E* (exposed) and *I* (infectious). The gonotrophic cycle is N_G stages long, with a temperature-dependent number of stages progress per day, $gprog$ (Table 2A). Mosquitoes bite when *g=0*. The extrinsic incubation period is N_G stages long, and for LRVF a fixed amount of incubation occurs each day. The time-varying probabilities $P_{surv}^{t,z}$ (daily survival probability, temperature-dependent) and $P_{infect}^{t,z}$ (daily infection probability from livestock hosts) are given in Table 2A.

The dynamics of the gonotrophic cycle, extrinsic incubation, application of daily survival and infection are applied by considering different categories of the adult mosquito population at time *t+1* relative to that at time *t*, as follows:

1. Susceptible biting mosquitoes remaining uninfected:

$$Z_S^{t+1,gprog} = P_{surv}^{t,z}.\left(1-P_{infect}^{t,z}\right).Z_S^{t,0}$$

2. Susceptible biting mosquitoes becoming exposed:

$$Z_E^{t+1,gprog,sprog} = P_{surv}^{t,z}.P_{infect}^{t,z} Z_S^{t,0} \qquad 0 \le sprog < N_E$$

$$Z_E^{t+1,gprog,N_E-1} = P_{surv}^{t,z}.P_{infect}^{t,z} Z_S^{t,0} \qquad otherwise$$

(Newly infected mosquitoes are forced to spend one day in the last element of the exposed class if $sprog > N_E-1$.)

3. Gonotrophic development of other mosquitoes in category *S:*

$$Z_S^{t+1,ig+gprog} = P_{surv}^{t,z}.Z_S^{t,ig} \qquad ig+gprog < N_G, \qquad ig = 1, N_G-1$$

$$Z_S^{t+1,0} = P_{surv}^{t,z}.Z_S^{t,ig} \qquad otherwise, \qquad ig = 1, N_G-1$$

4. Larval maturation (see Immature mosquito section):

$$Z_S^{t+1,1} = Z_S^{t+1,1} + \; P_{larvsurv}^{t,z} L^{t,iL} \qquad iL = N_L - 1$$

5. Development (gonotrophic and incubation) of mosquitoes in categories *E* and *I:*

$$Z_E^{t+1,ig+gprog,is+sprog} = P_{surv}^{t,z}.Z_E^{t,ig,is} \qquad ig + gprog < N_G, is + sprog < N_E, \quad ig = 0, N_G - 1,\ \ is = 0, N_E - 1$$

$$Z_E^{t+1,0,is+sprog} = P_{surv}^{t,z}.Z_E^{t,ig,is} \qquad ig + gprog \geq N_G, is + sprog < N_E, \quad ig = 0, N_G - 1,\ \ is = 0, N_E - 1$$

$$Z_I^{t+1,ig+gprog} = P_{surv}^{t,z}.Z_E^{t,ig,is} + P_{surv}^{t,z}.Z_I^{t,ig} \qquad ig + gprog < N_G, is + sprog \geq N_E, \quad ig = 0, N_G - 1, is = 0, N_E - 1$$

$$Z_I^{t+1,0} = P_{surv}^{t,z}.Z_E^{t,ig,is} + P_{surv}^{t,z}.Z_I^{t,ig} \qquad ig + gprog \geq N_G, is + sprog \geq N_E, \quad ig = 0, N_G - 1, is = 0, N_E - 1$$

Immature vectors

Immature mosquitoes are separated into infection categories *x=S* and *x=I* if transovarial transmission is set, otherwise the egg and larval populations *E* and *L* are all considered uninfected.

Ovipositioning is via laying factor B^t. Egg and larval survival probabilities $P_{eggsurv}$ and $P_{larvsurv}^{t,z}$ are predefined/precalculated for time t respectively (Table 2A). Vertical transmission for Aedes eggs occurs at a rate φ. *For Culex,* $\varphi=0$.

For Aedes eggs *E*, with vertical transmission:

$$E_S^{t+1,0} = \mathrm{B}^t\left(Z_S^{t,0} + Z_E^{t,0} + (1-\varphi)Z_I^{t,0}\right) \qquad Z^{t,0} < Z_{cap}$$

$$E_S^{t+1,0} = \frac{\mathrm{B}^t Z_{cap}}{Z^{t,0}}\left(Z_S^{t,0} + Z_E^{t,0} + (1-\varphi)Z_I^{t,0}\right) \qquad otherwise$$

$$E_I^{t+1,0} = \mathrm{B}^t\varphi Z_I^{t,0} \qquad Z^{t,0} < Z_{cap}$$

$$E_I^{t+1,0} = \frac{\mathrm{B}^t Z_{cap}\varphi Z_I^{t,0}}{Z^{t,0}} \qquad otherwise$$

Aedes eggs undergo a separate drying stage of length N_E days. Rainfall averages Δ_d^t and Δ_w^t over drying and wetting periods τ_d and τ_w are compared to "trigger" thresholds θ_{dry} and θ_{wet} to determine if the drying and wetting conditions (respectively) have been met for a given day. If the drying condition is not met, eggs at all drying "development" stages, *iE*, are reset to stage zero. If the wetting condition is not met, fully dry "mature" eggs remain at stage N_E and do not hatch.

For infection category x:

$$E_x^{t+1,iE+1} = P_{eggsurv} E_x^{t,iE} \qquad iE = 0, N_E - 2;\ \ \Delta_d^t \leq \theta_{dry}$$

(drying condition met)

$$E_x^{t+1,0} = P_{eggsurv} \sum E_x^{t,iE} \qquad iE = 0, N_E - 2;\ \Delta_d^t \leq \theta_{dry}$$

(drying condition not met – egg drying is reset)

Eggs which have completed the drying stage and are waiting to hatch:

$$E_x^{t+1,iE+1} = P_{eggsurv} E_x^{t,iE} + P_{eggsurv}^{t,z} E_x^{t,iE+1} \quad iE = N_E - 1;\quad \Delta_d^t \leq \theta_{dry}\ ;\ \Delta_w^t \leq \theta_{wet}$$

(Drying condition met; wetting condition not met this timestep so eggs remain in stage N_E. Second term is to account for eggs remaining in stage N_E if wetting condition not met the previous day.)

$$E_x^{t+1,iE+1} = 0 \qquad iE = N_E - 1;\quad \Delta_d^t \leq \theta_{dry}\ ;\ \ \Delta_w^t > \theta_{wet}$$

(Drying condition met; wetting condition met so eggs hatch and contents of box N_E move to larval stage.)

Following hatching, larval development proceeds at a constant rate with mature vectors emerging after N_L days. For Culex mosquitoes, immature development is entirely contained within the "larval" stage.

Aedes hatching:

$$L_x^{t+1,0} = P_{eggsurv} E_x^{t,iE} + P_{eggsurv}^{t,z} E_x^{t,iE+1} \qquad iE = N_E - 1; \Delta_d^t \leq \theta_{dry}\ ;\ \ \Delta_w^t > \theta_{wet}$$

Culex larvae:

$$L_x^{t+1,0} = E_x^{t+1,0}$$

Larval development (both species)

$$L_x^{t+1,iL+1} = P_{larvsurv}^{t,z} L_x^{t,iL} \qquad iL = 0, N_L - 2$$

$$L_x^{t+1,iL+1} = 0 \qquad otherwise$$

(matured larvae have entered the adult stage)

References

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